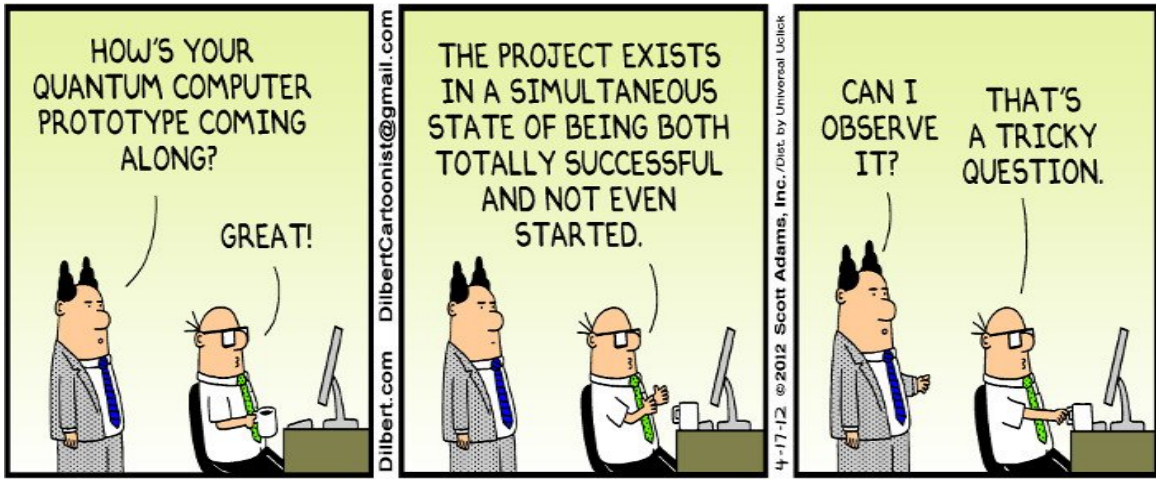
Design and Operation of the Trapped Ion Quantum Computer (TIQC)



Special Topics in Computer Science: Quantum Computing

CSC591/ECE592 – Fall 2018

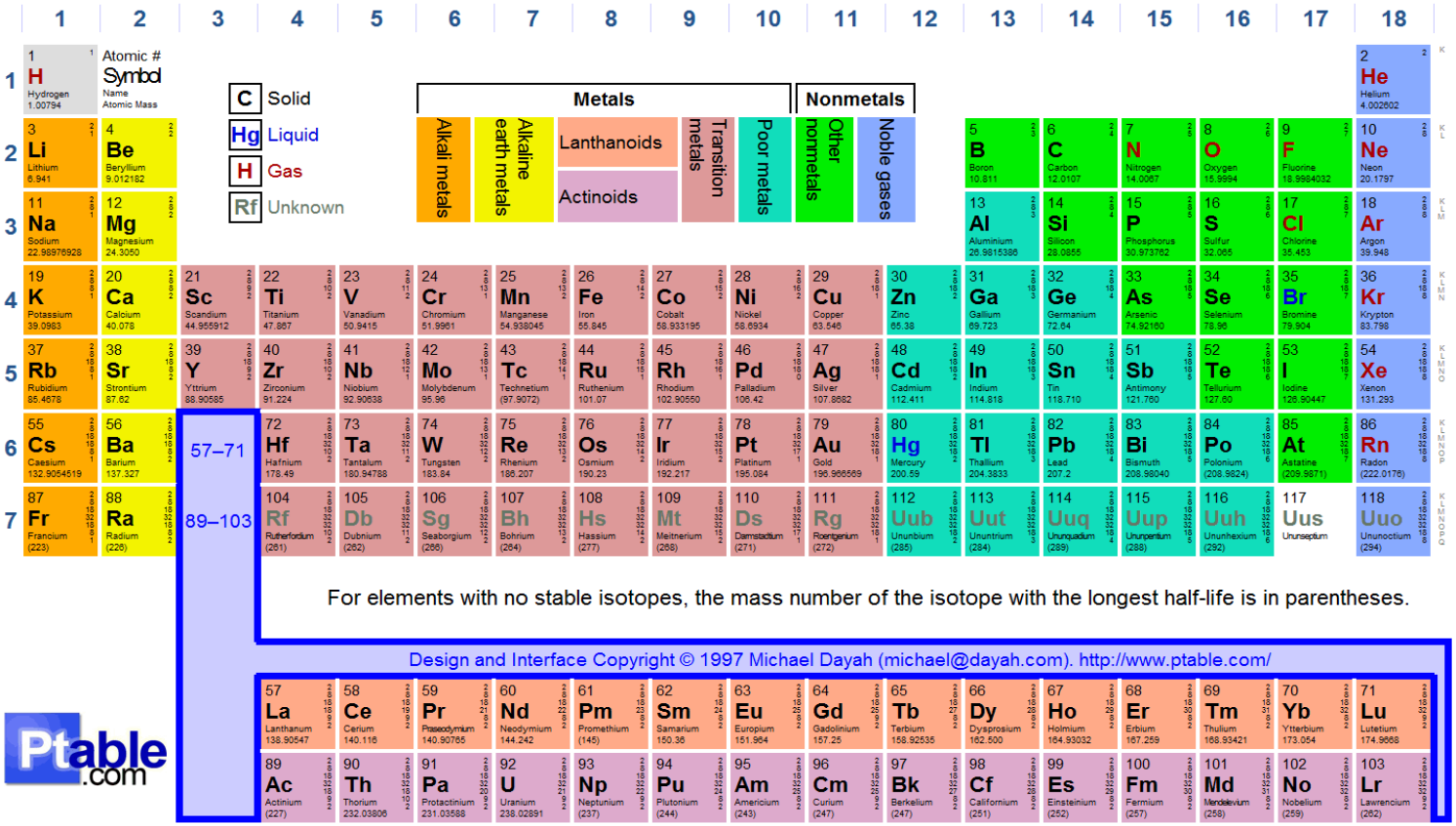
Outline the Design Requirement for a TIQC

- The TIQC must be constructed such that
  1. Selected materials can emulate 1 and 2 qubit operations
  2. Can construct universal quantum gates without collapsing the entire quantum computing computation
  3. The user must be able to extract a final measurement from the state of the qubits at the conclusion of the quantum computing program
  4. System must be scalable

Step 1.
Select Materials That Can Emulate One and Two Qubit Operations

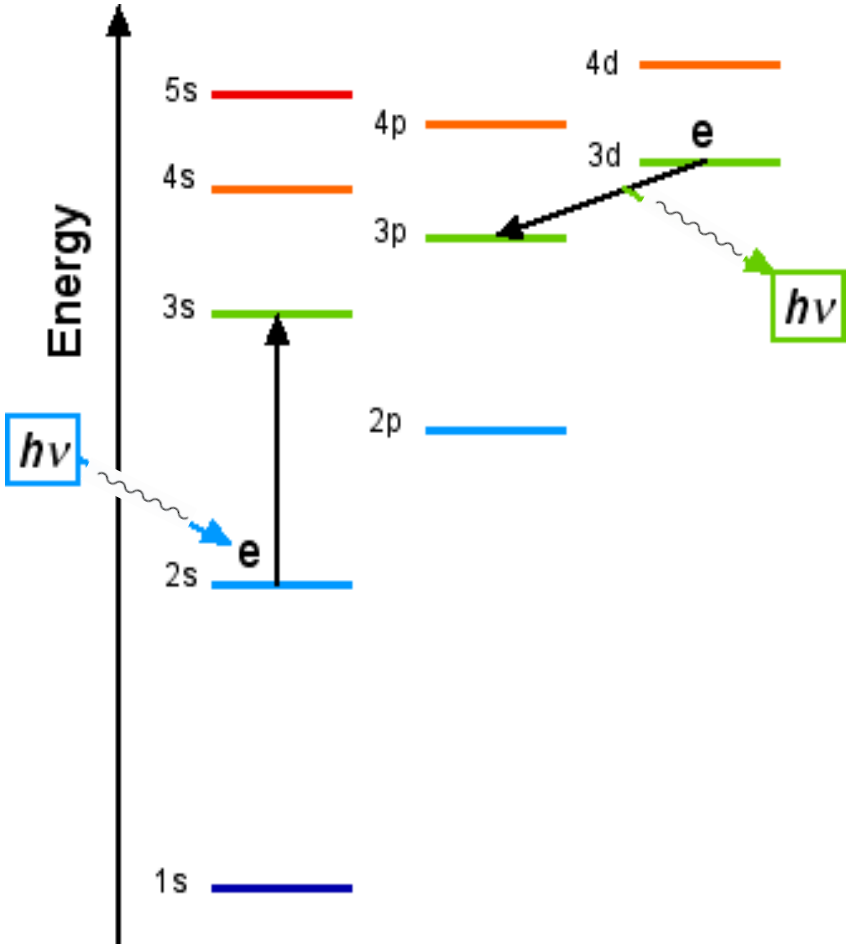
Start by Selecting a Material for the TIQC

Periodic Table of Elements



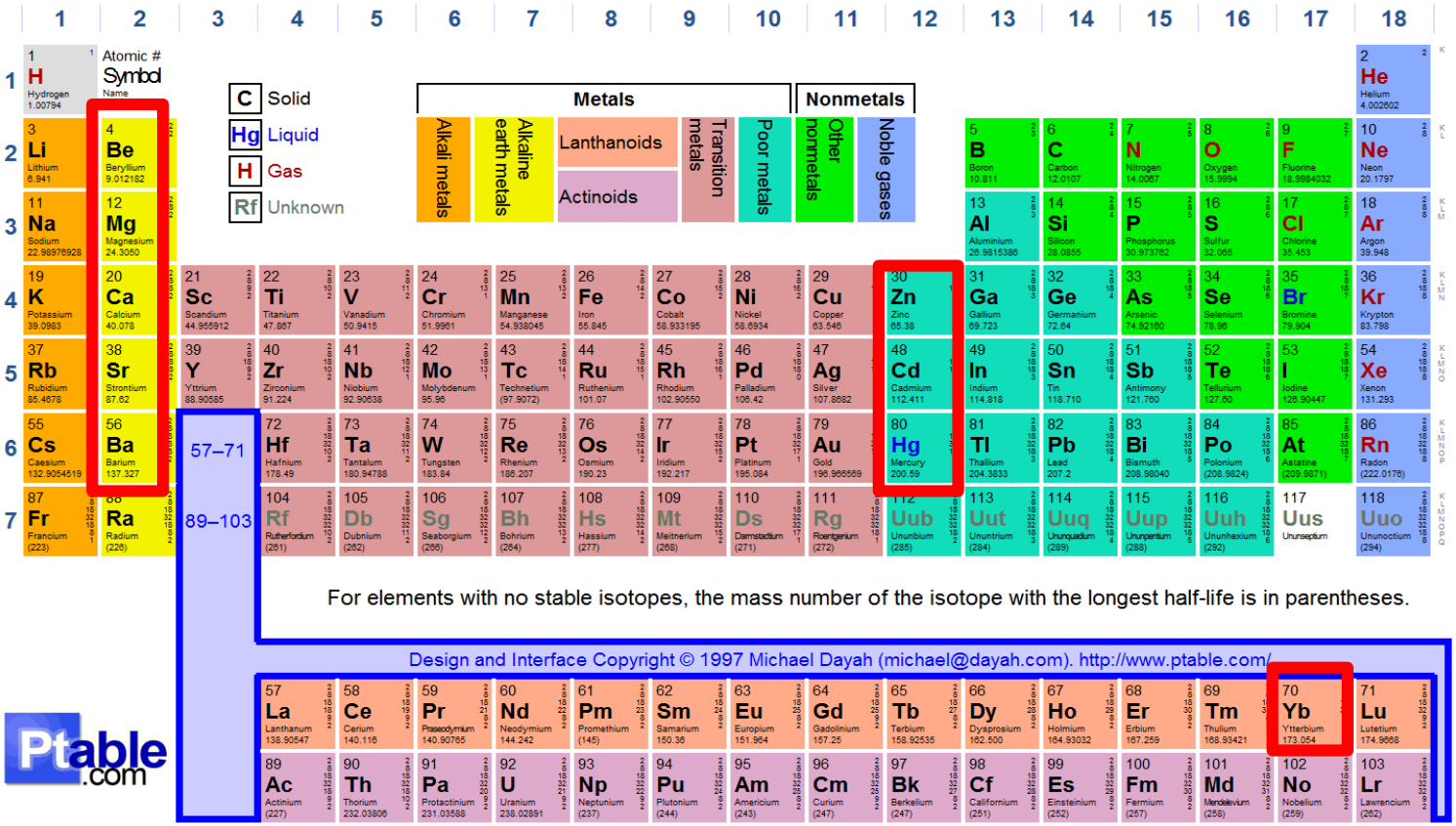
QM Describes Each Element's Atomic Structure (Energy Levels and Transitions)

- Electrons can change energy states by transitioning among different quantized energy levels
- Electrons absorb and emit discrete quantities of energy and angular momentum when undergoing these transitions

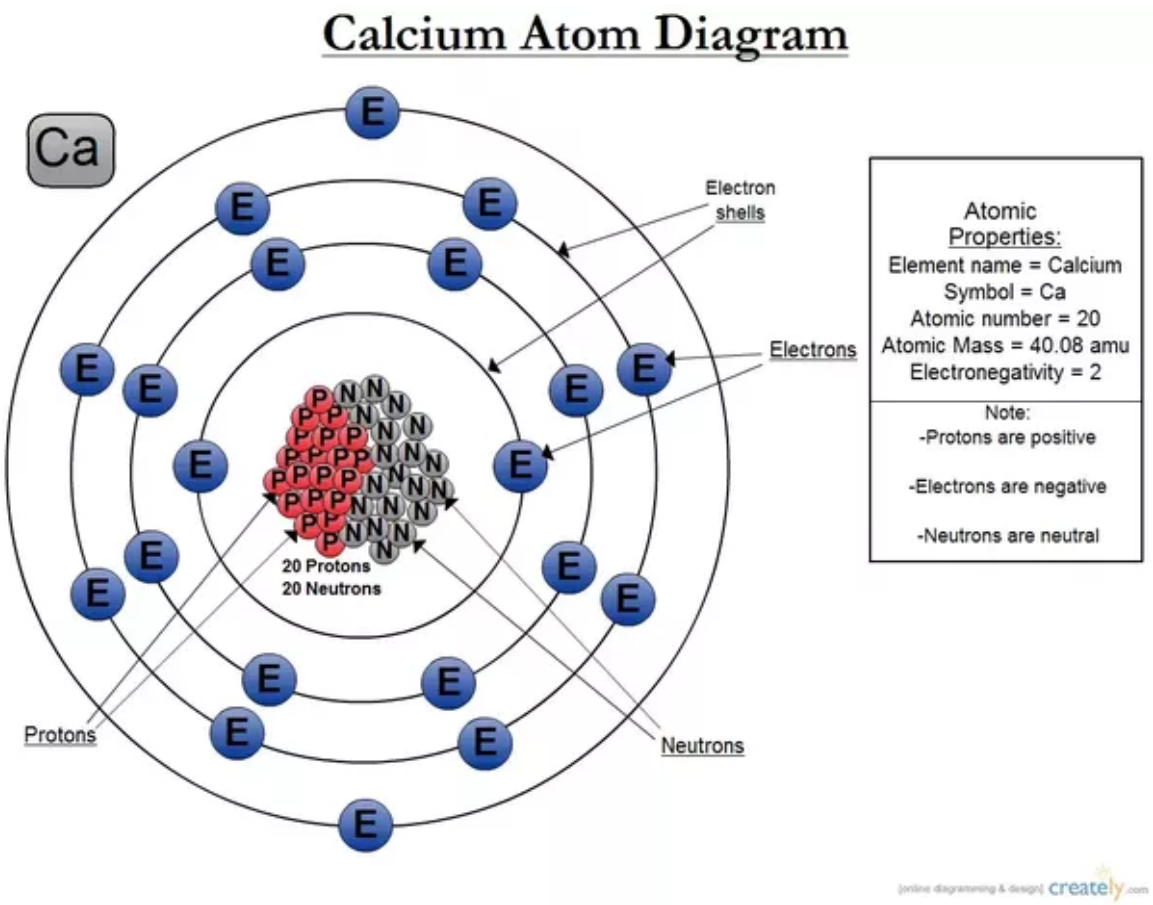


Start by Selecting a Material for the TIQC

Periodic Table of Elements



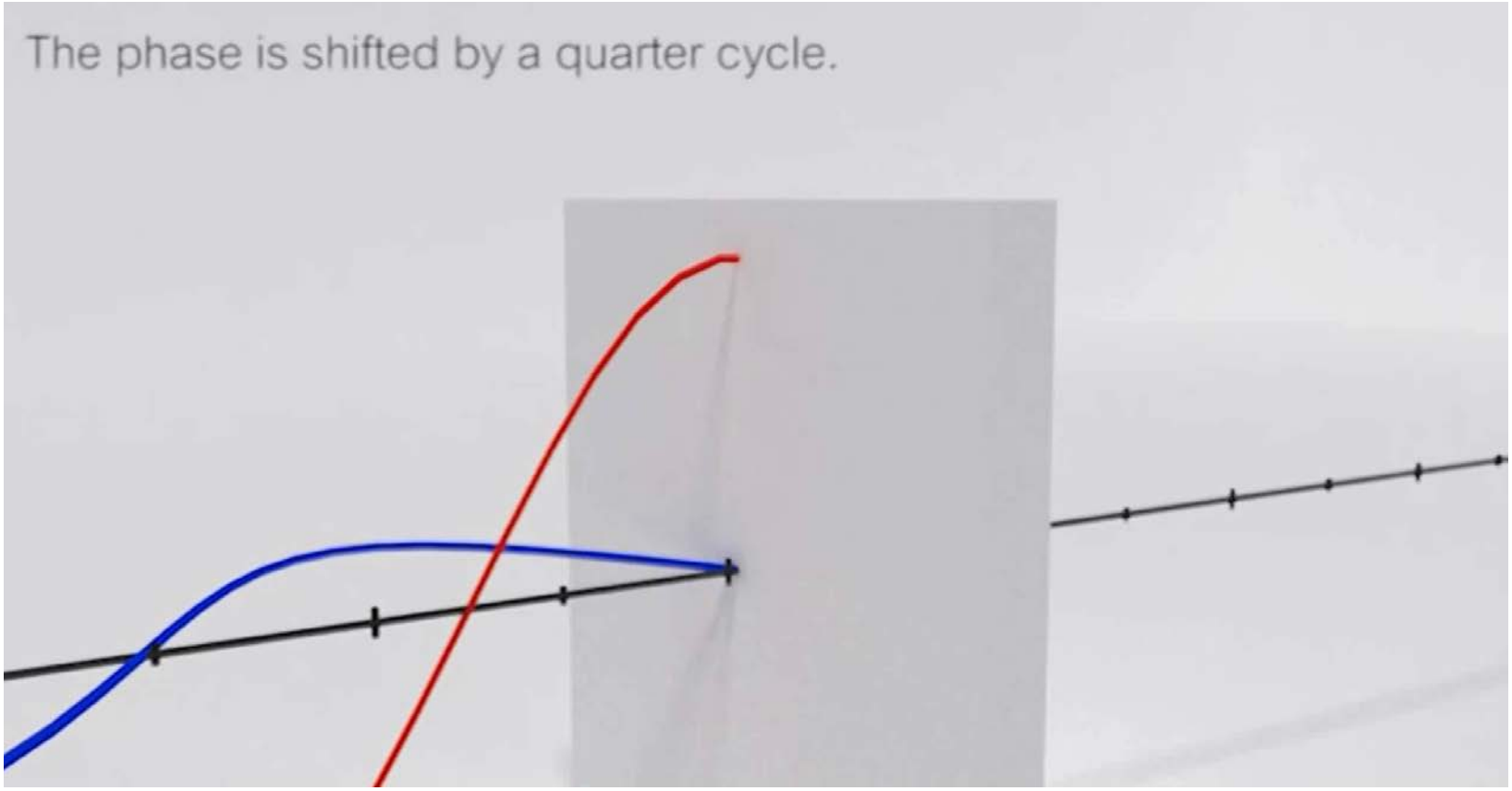
Select the Calcium Atom (^{40}Ca)



Select Mechanism to Interact with the Selected Material

- Electromagnetic fields are a primary source for transferring energy and angular momentum to electrons in the ^{40}Ca atom via electromagnetic force
- Bound state electrons in an atom will absorb and emit discrete quantities of energy and units of angular momentum determined by
  - Difference between the two bound state energy levels
  - The initial and final total angular momentum (combination of both the electron's orbital angular momentum and an "internal" angular momentum called "spin")

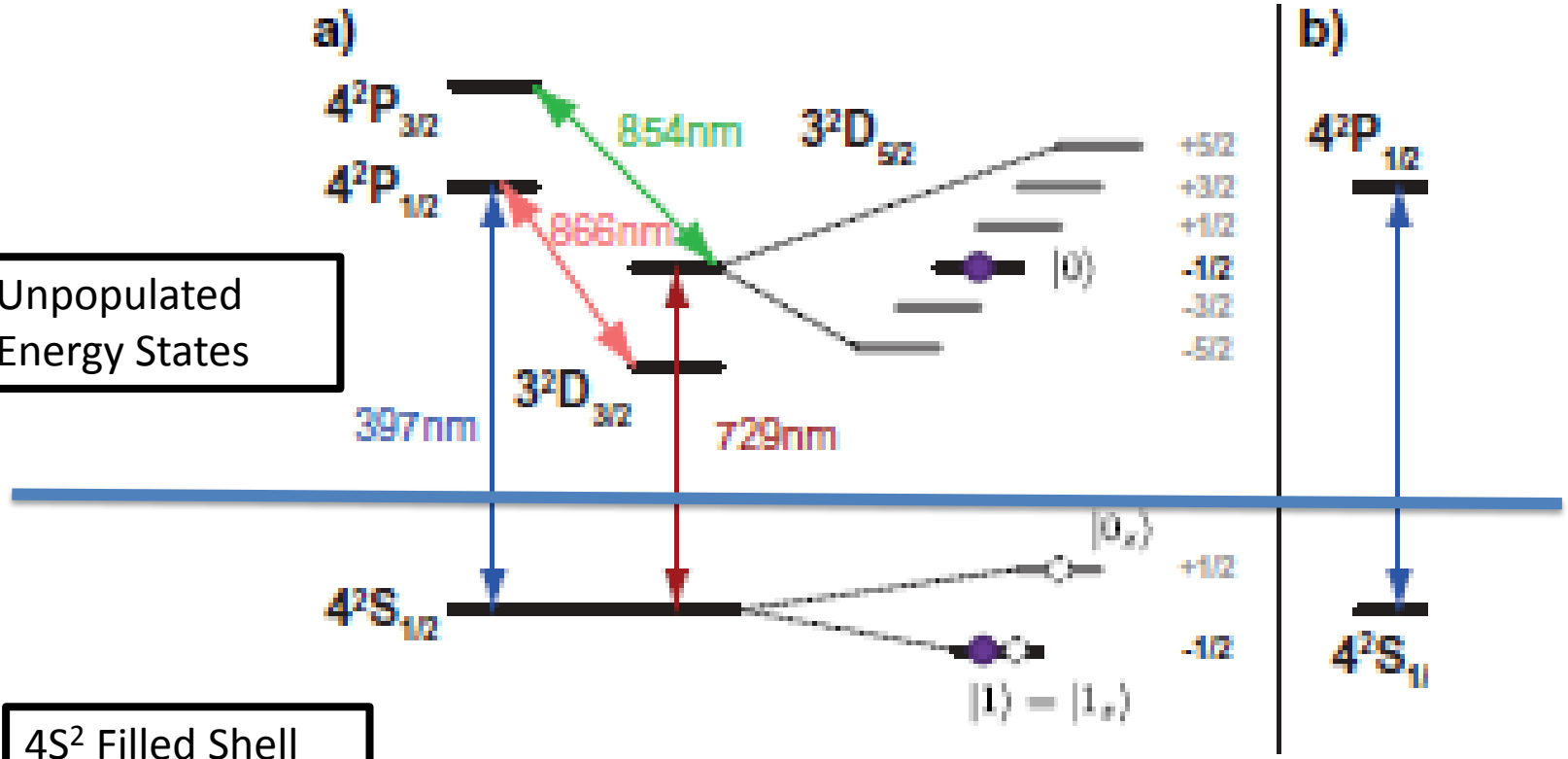
Propagation of Electromagnetic Fields



Transferring Energy to/from Bound State Electrons in a Material

- By selecting a specific wavelength of electromagnetic radiation the experimentalist can control the
  - Energy absorbed or emitted by the electron
  - Discrete units of angular momentum transferred
- There are specific “quantum mechanics” rules constraining transitions between energy levels based on the transition energy and change in angular momentum (Selection Rules)

Focus on the Atomic Spectra of ^{40}Ca



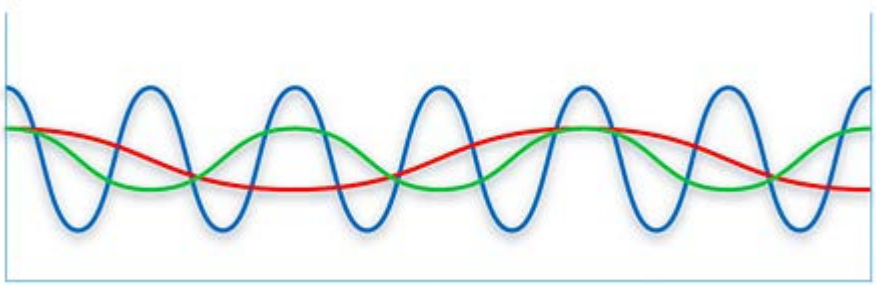
Unpopulated Energy States

$4S^2$ Filled Shell Energy States

Lasers

Electromagnetic Radiation Properties

- Light is composed of many electromagnetic fields of many different energies (frequencies)



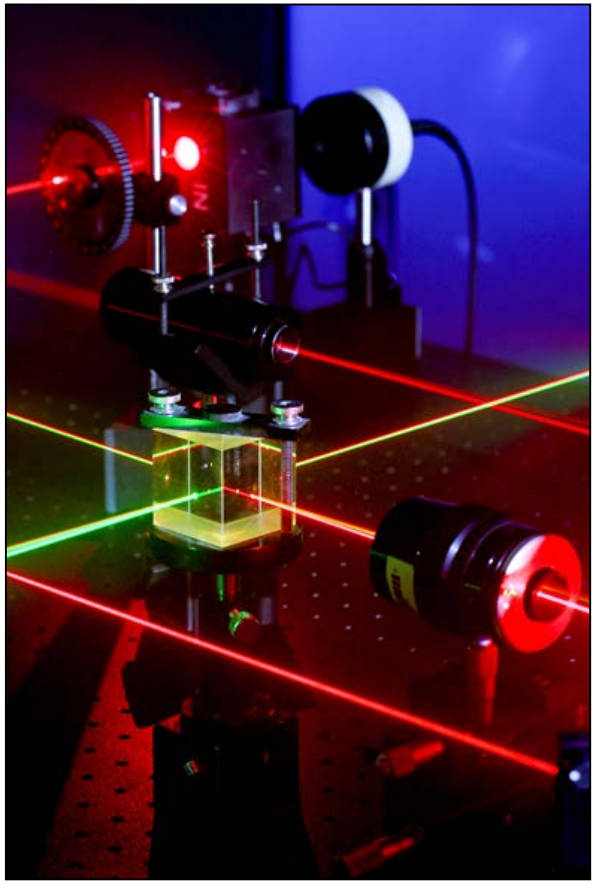
Incoherent Light

- Need light with properties of coherence (light with specific frequency and common phase)

Coherent Light

Need a Focused Source of Energy - Lasers -

- Lasers (coherent light source) allow experimentalists to “dial-up” a specific wavelength that will cause the electron to transition (resonate) between two different energy levels



Lasers in the Experimental Apparatus

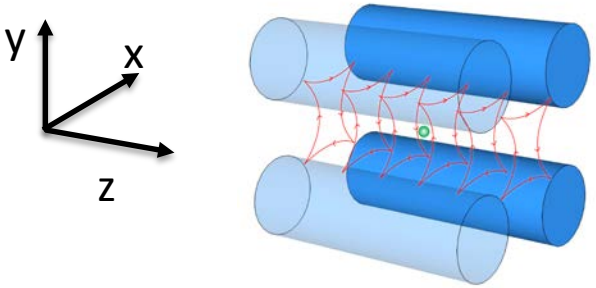
- By varying the laser's
  - Polarization
  - Wavelength
  - Duration of the laser light pulse

  the behavior of the electron can be controlled
- From a quantum computing perspective this is an effective mechanism for creating rotations and transformations

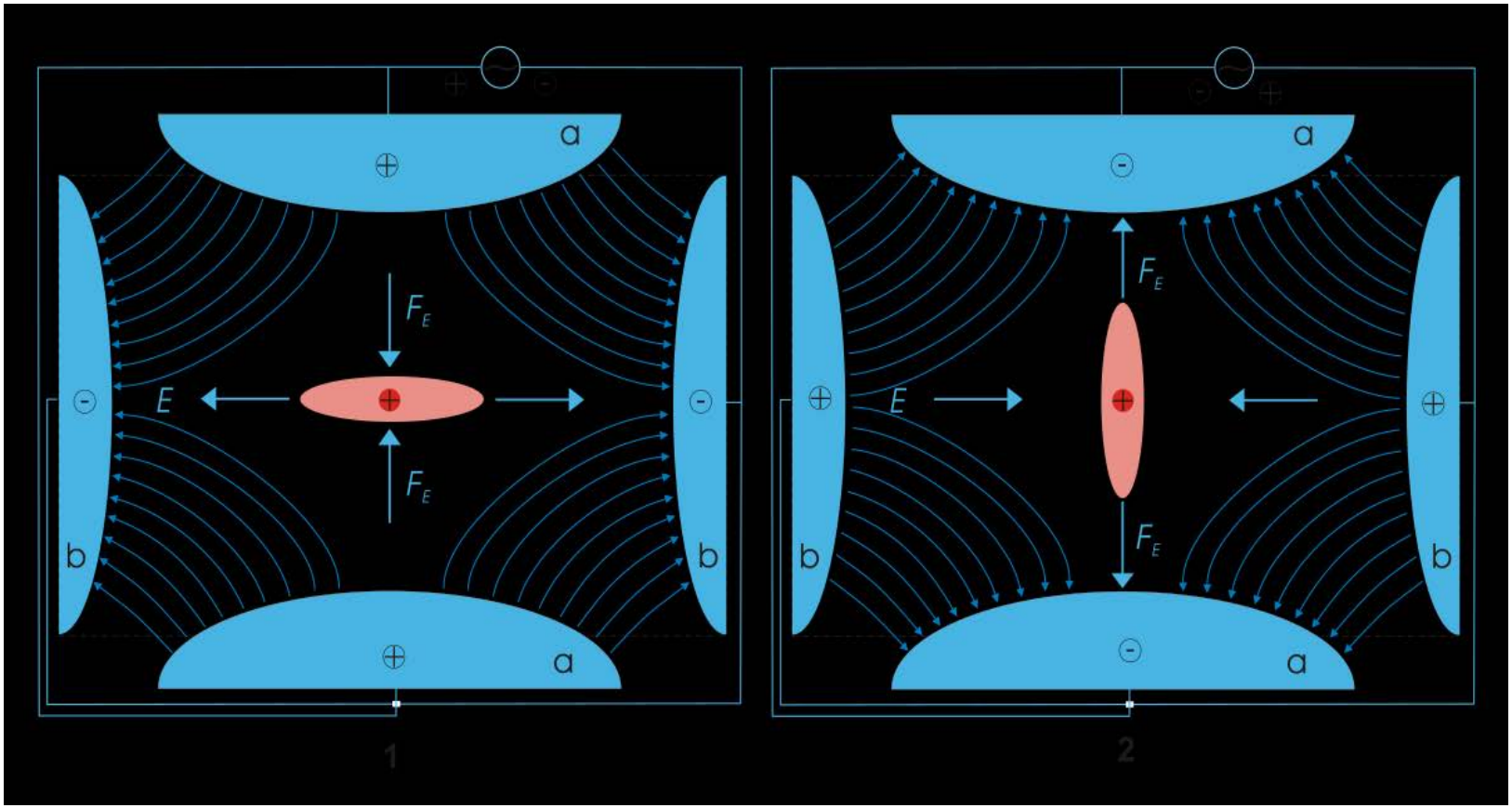
Construct the TIQC Experimental Apparatus

Confine the Atoms into a "Device" Carl Friedrich Gauss's Objection

- Static electric field confinement of the atoms in three dimensions is not possible
- Div **E** = 0 → no net inward force to constrain motion of the atoms
- force cannot be inward in all directions → at least one direction where ions can escape

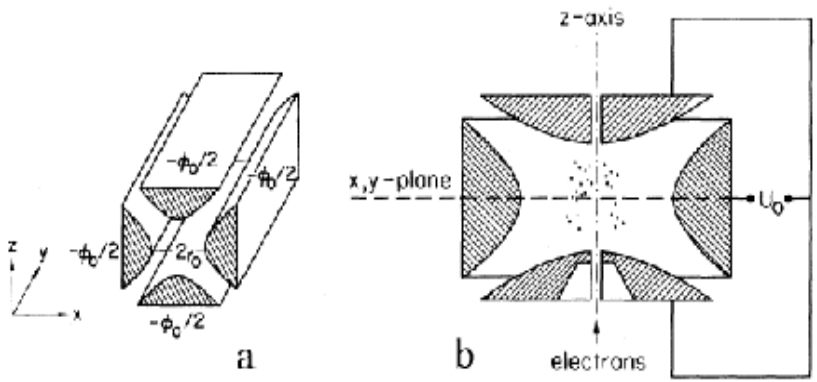


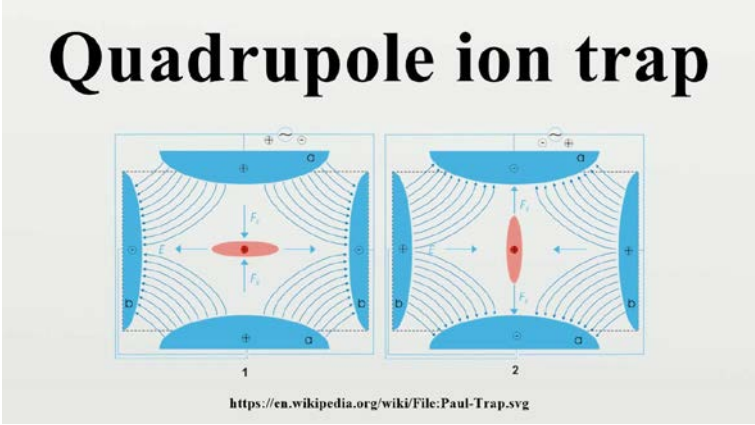
Consider Quadrapole Field



Put the Ca Atoms into “Trap” Apparatus

- Construct an apparatus that will confine ions along one dimension
- Consider a static quadrapole field



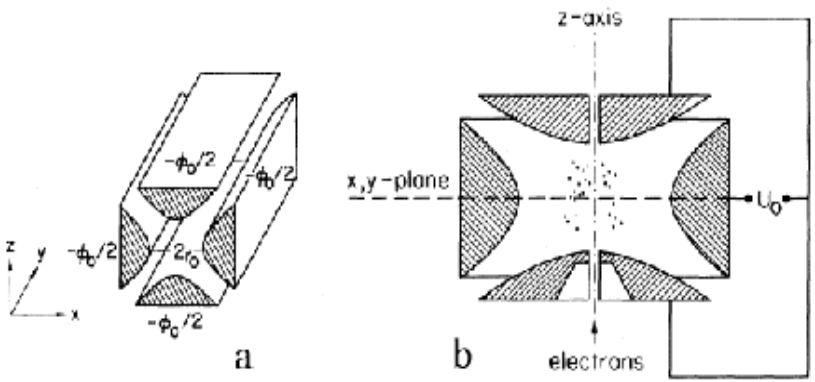


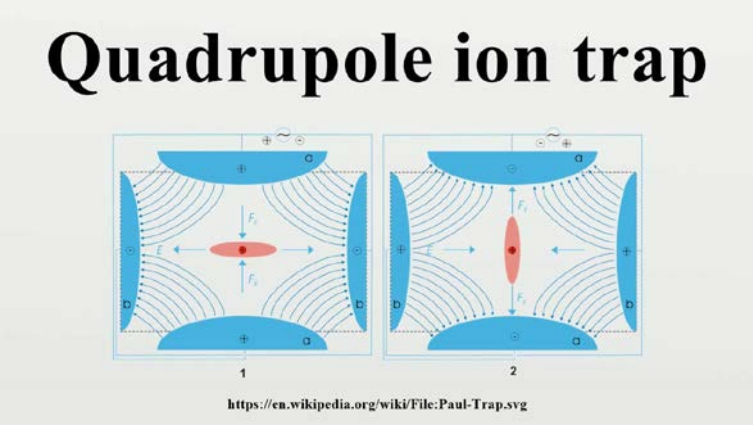
Model an Ion in a Stationary Quadrupole Field*

* You tube video (stationary saddle) https://www.youtube.com/watch?v=XTJznUkAmIY

Put the Ca Atoms into “Trap” Apparatus

- Modify the stationary quadrapole field
- Make a periodic rotation of the shape of the field lines as seen by the ion by applying an RF voltage
- In addition, the ends of the cylinders are biased at different dc voltages from the cylinder center so that the charged ions are axially confined



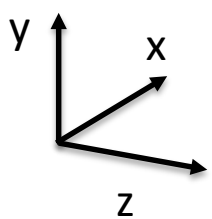


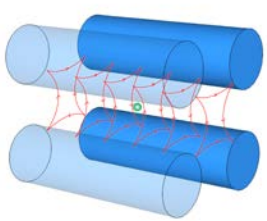
Rotating Saddle Point Surface*

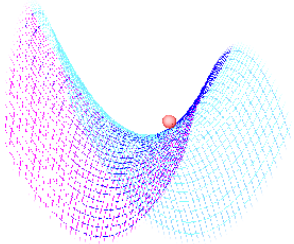
* You tube video https://www.youtube.com/watch?v=rJ13qwRYs

Additional Effect of the Periodic RF Potential

- Net effect produces a combined effect
  - The combination of the RF and DC voltages also produce a harmonic potential
  - The electrostatic repulsion of each ion creates a string of ions trapped along the z-axis of the trap
  - Under these conditions the motion of the confined ions becomes quantized as a 1-dimensional harmonic oscillator with equally spaced energy levels $\hbar\omega$



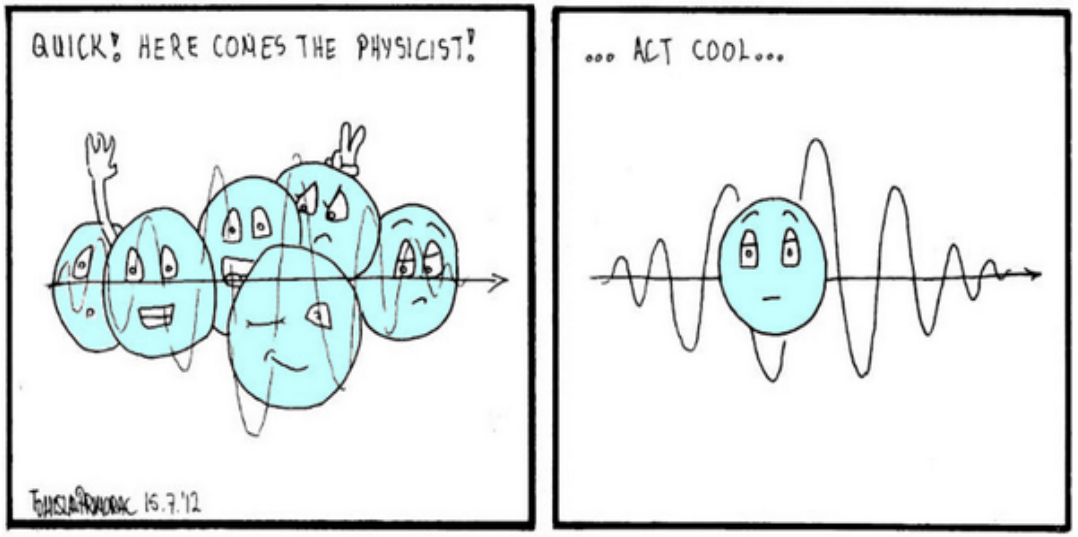




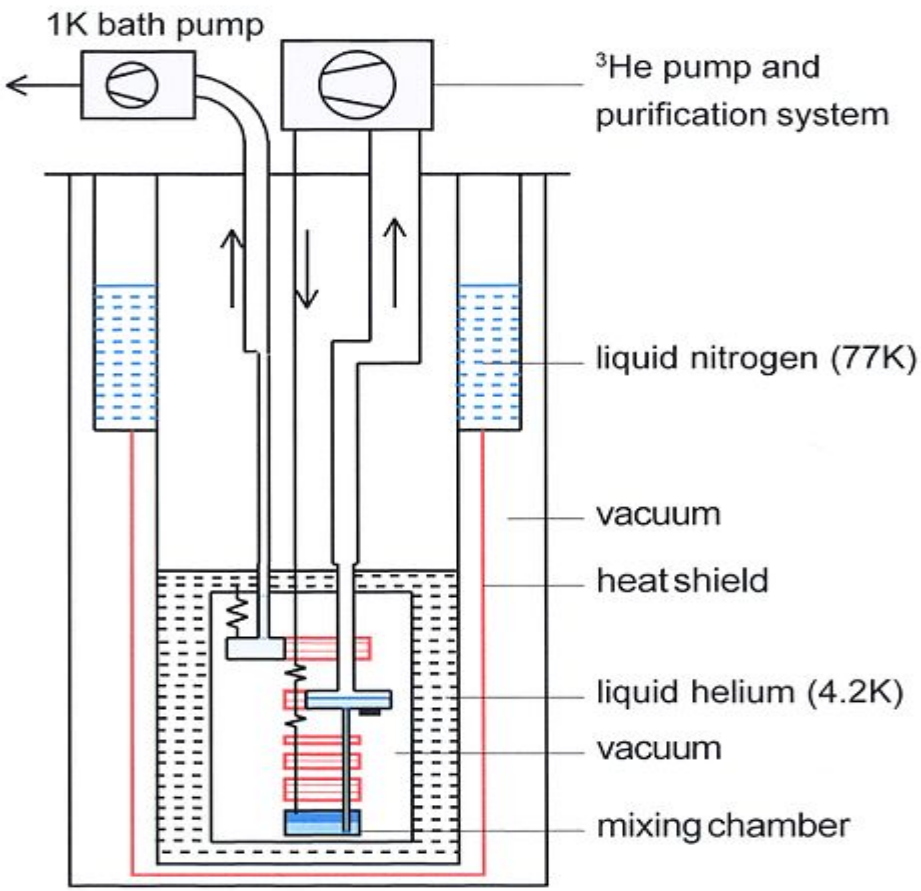
Low Temperature Requirement for the TIQC Apparatus

Low Temperature Requirement for the Experimental Apparatus

- Electrons are subject to many types of energy fluctuations at room temperature
- There are many excited states to which the electron can transition (unwanted volunteers)
- Suppress this “jitter” by cooling the material



Dilution Refrigerator *

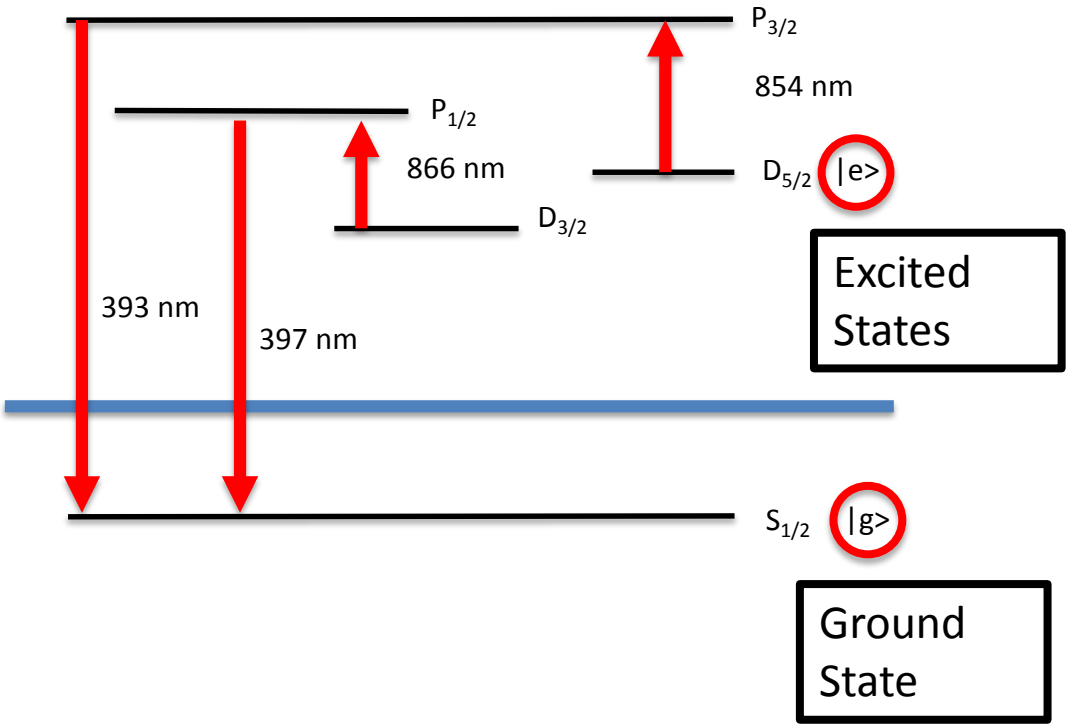


* Image from http://www.wikiwand.com/en/Dilution_refrigerator

Low Temperature Experimental Apparatus IBM Q Quantum Computer Cryostat

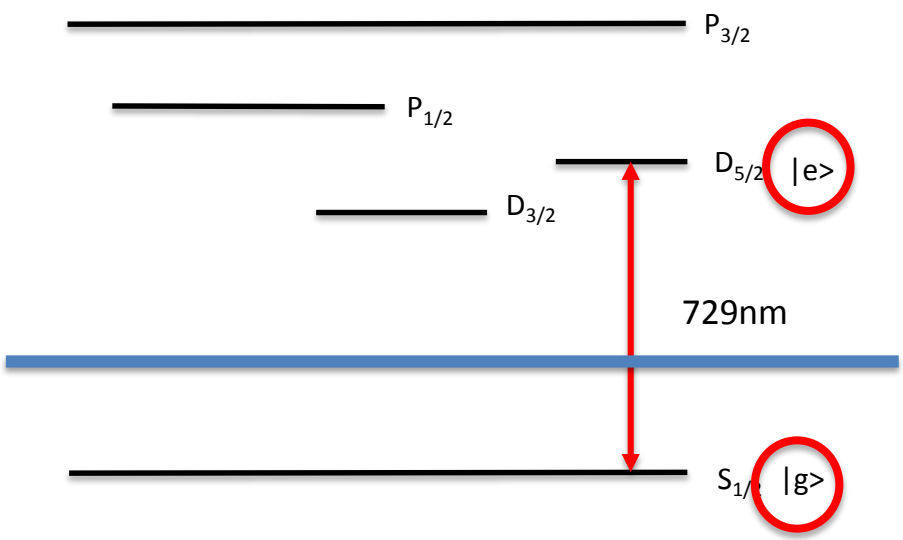
Manipulating the Ground and Excited States of the Electrons in the Ca40 atom

- Cool the Apparatus to Limit the Size of the Available Hilbert Space
- Need multiple lasers tuned to specific wavelengths to depopulate the unwanted excited states



Building a Long Lived Ion Qubit State

- Want to identify an excited state that will be "long-lived"
- From laws of QM this is a forbidden transition and so the excited state will be long lived (~1 sec) compared to the lifetime of an allowed transition (~ 1 nanosecond)
- This transition can be identified as a potential candidate for a stable qubit

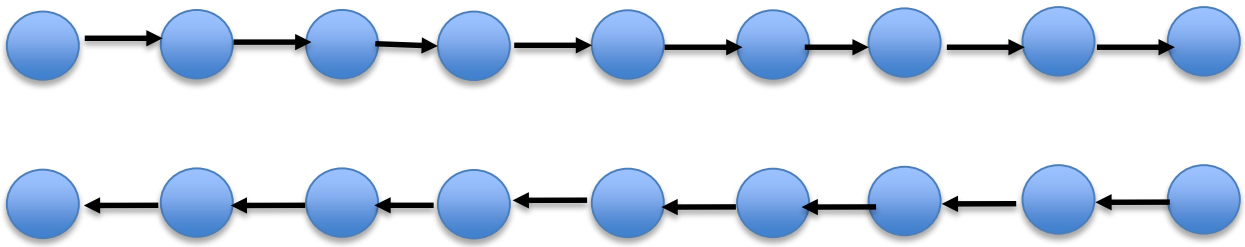


Low Temperature Impact on the ^{40}Ca Electron's Vibrational Energy States

- Apparatus operates near absolute zero (15 mK) such that
  1. Only a few excited states above the ground state are accessible to the ion (this limits the size of the Hilbert space available for energy transitions)
  2. The trap forms a 1D harmonic oscillator potential that stores the ^{40}Ca ions
  3. The ^{40}Ca ions can exhibit lowest level vibrational states (phonons) in the 1 dimensional harmonic oscillator potential when sufficiently cooled

Phonons

- Phonons in this context are center of mass energy eigenstates that represent the coupled vibrational modes of the entire lattice of ions



- These ions are at room temp and have many thermal vibrational modes that have been suppressed by cooling the material

Summary of TIQC Device Properties

- Have constructed a 2 level spin system interacting with an electromagnetic field
- Spin is physically confined within a 1-dimensional harmonic oscillator potential
- Spin interactions controlled by rotations in response to a laser pulse
- States are quantized with energy of scale $h\nu$
- These harmonic oscillator bound states are identified as center of mass phonon vibrations

Step 2.
Construct Universal Quantum Gates Without Collapsing The Entire Quantum Computing Computation

Identify Candidates for a Two Qubit System

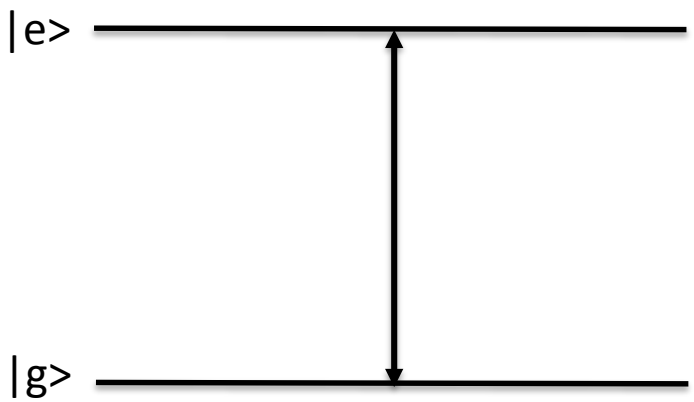
- Identify a single 2 level spin system interacting with an electromagnetic field
  a) Qubit can be identified by the quantized bound states of an atomic material as seen through ability of a spin to respond to an electromagnetic field
  b) A second qubit can be identified through the set of interactions of the ion's vibrational modes

$$\nu$$

Outer Product Representation of Available Qubit Quantum States in a TIQC

Two level Ion

Harmonic Potential Trap



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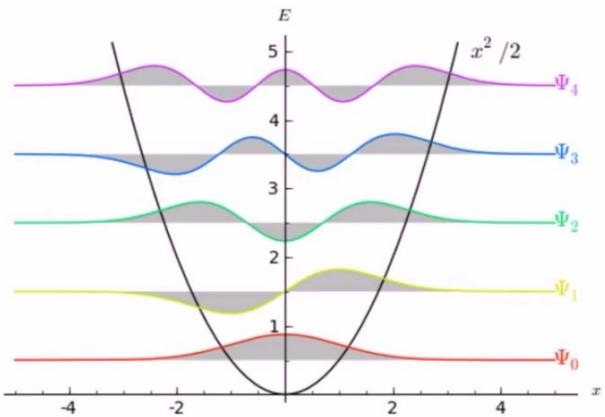
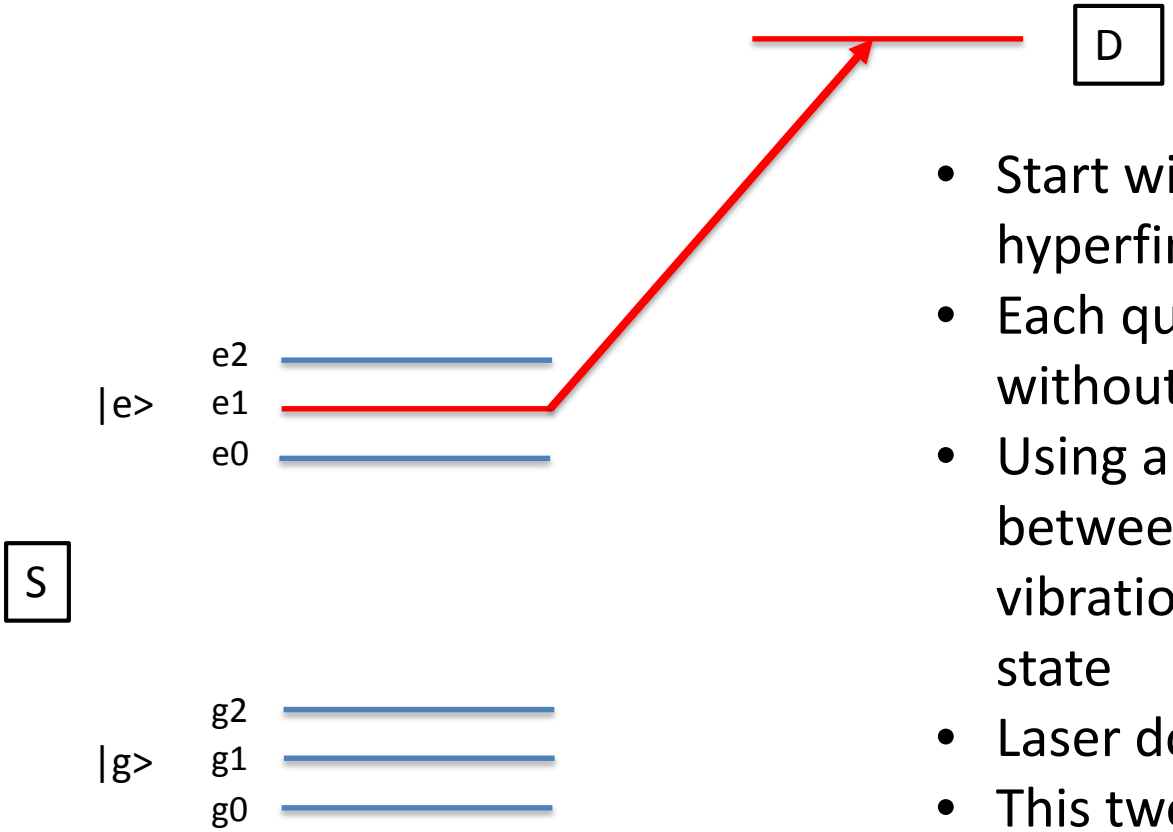
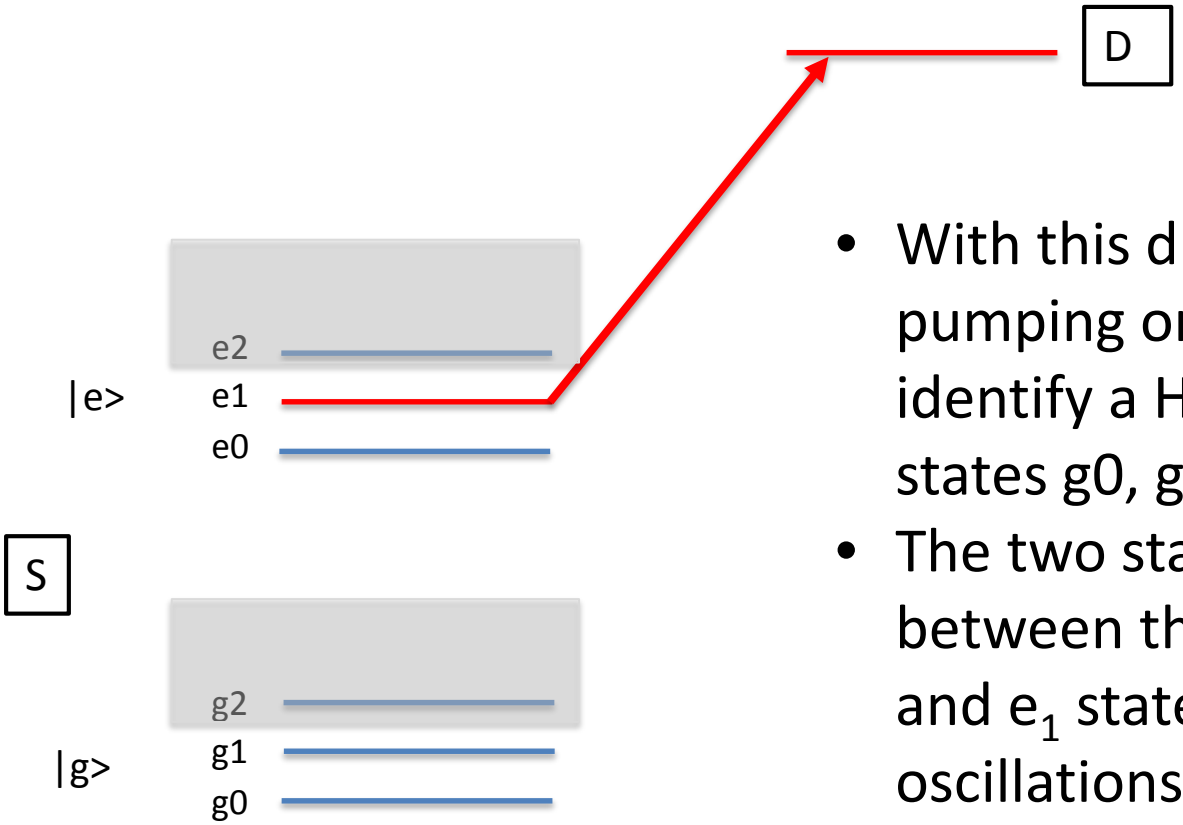


Single Ion Behavior



- Start with ion in an S state with 2 hyperfine states
- Each qubit has |g> and |e> without center of mass motion
- Using a laser select resonance between the |e1> excited vibrational state of |e> to a D state
- Laser does not affect g_0, g_1 or e_1
- This two state laser driven pulse produces Rabi oscillations

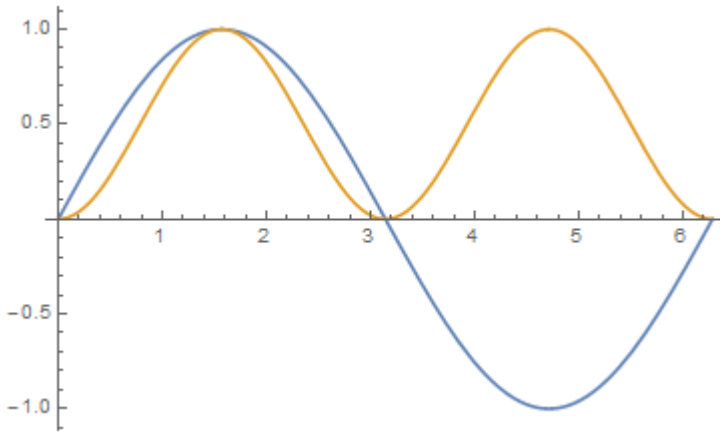
Construct a Phase Gate From Single Ion



- With this driven laser pulse pumping only this transition identify a Hilbert space with states g0, g1, e0, e1
- The two state oscillation between the auxiliary D state and e_1 state produce Rabi oscillations

Rabi Oscillations

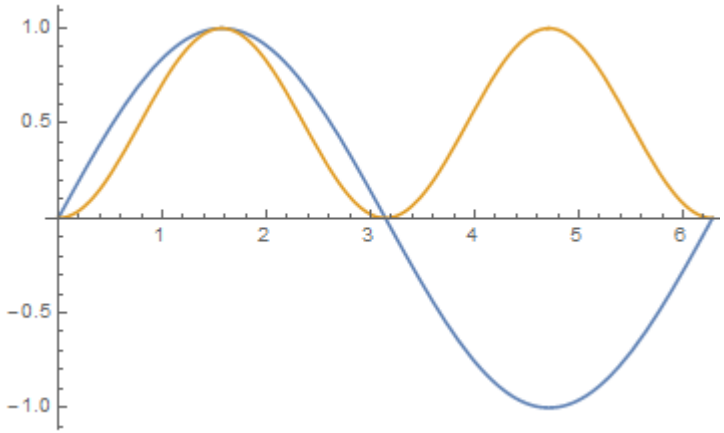
- Rabi oscillations (also known as the Rabi cycle or Rabi flop) is the cyclic behavior of a two-level quantum system in the presence of an oscillatory driving field (such as a laser pulse)
- Figure below shows cyclic probability amplitude (blue) and the measurement probability (yellow)



Rabi Oscillations Information Used to Create a Phase Gate

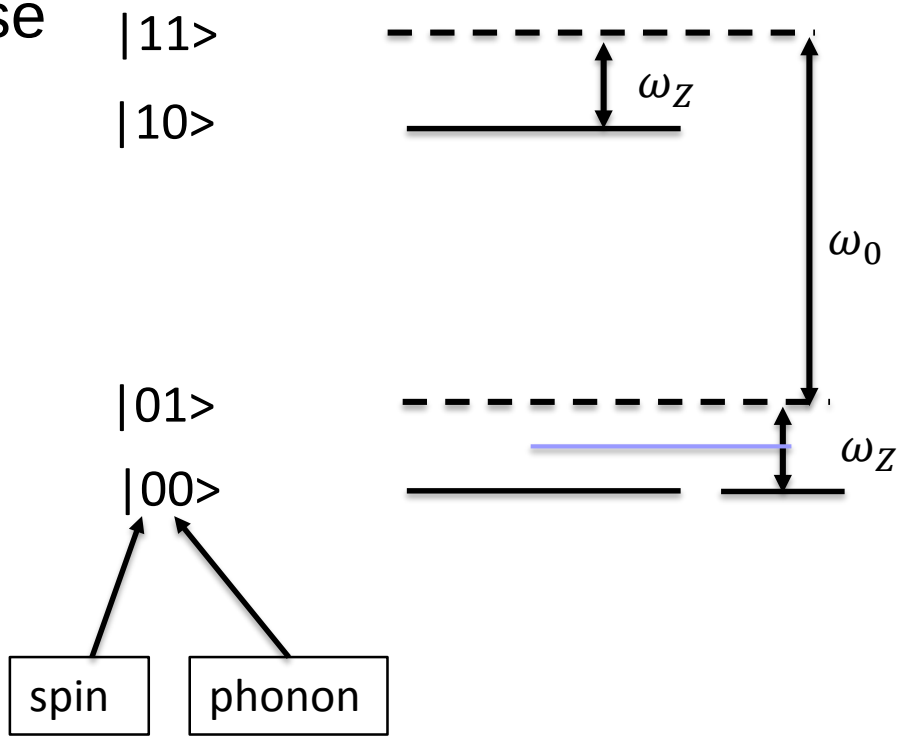
- Rabi oscillation after one period has changed the phase of the probability amplitude by π (phase is $-i$) (blue)
- Quantities measured in the lab are the probabilities (yellow)
- Figure shows that after the system has returned to the original state the probability has shifted by 2π but the phase by π ($-i$)
- <u>2π pulse in population shifts phase of wavefunction by π ($-i$)</u>

$g_0 \rightarrow g_0$
$g_1 \rightarrow g_1$
$e_0 \rightarrow e_0$
$e_1 \rightarrow -e_1$



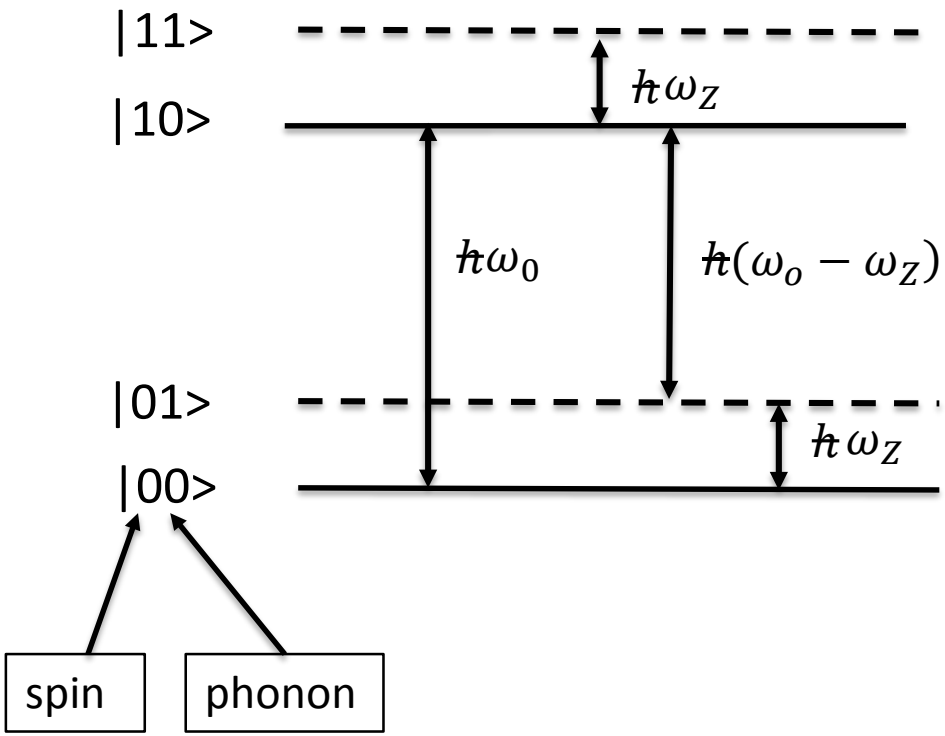
Phonon Vibration States for Single Atom

- Assume particle is cooled so that it is near its lowest vibrational state
- Have a ladder of these harmonic oscillator states



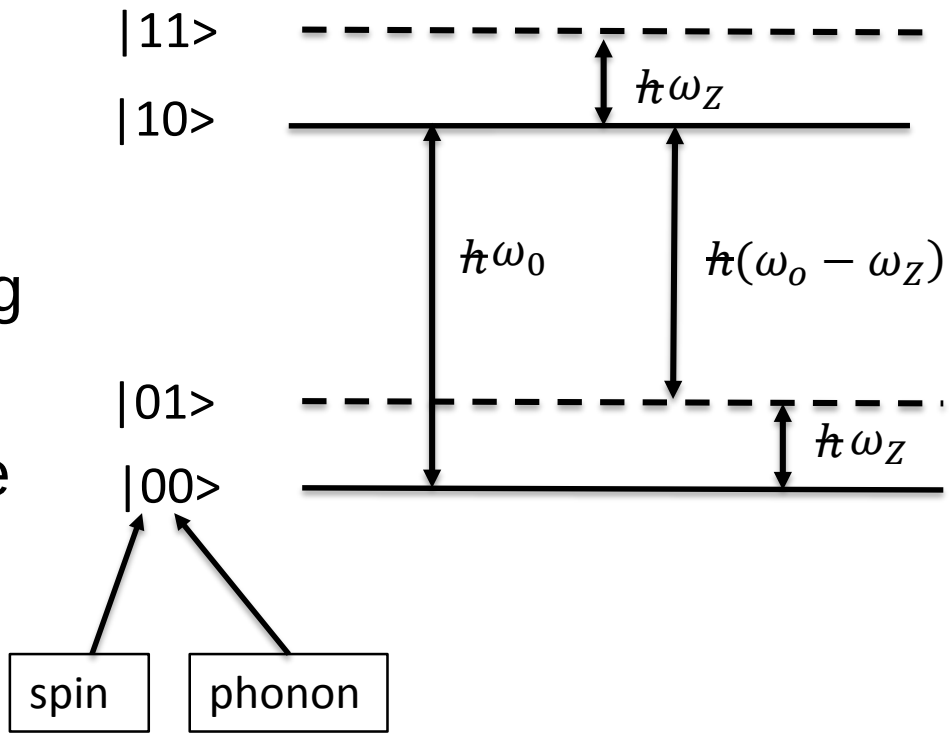
Energy Levels of a Single Atom

- Assume ion is cooled so that it is near its lowest vibrational state
- Have a ground and excited spin state and a ground and excited vibrational phonon state



Energy Levels of a Single Atom

- Use a laser detuned from the $\hbar\omega_0$ spin transition tuned to |10>→|01> transition energy $\hbar(\omega_o - \omega_Z)$
- Uniquely forces a transition from |10> → |01> without possibility of inducing other transitions
- This places the entire ion chain in the first excited vibrational state of spin |0>



Two ^{40}Ca Atoms

- Construct a set of basis vectors from a linear vector space describing wavefunction of two ions (A and B) and a collective phonon vibrational state

$$|0_A > |0_B > |0 >$$

$$|0_A > |1_B > |0 >$$

$$|1_A > |0_B > |0 >$$

$$|1_A > |1_B > |0 >$$

Laser Pulse Generates a π Rotation Pulse Directed to Ion A

- Select two ^{40}Ca ions (A and B) and the collective phonon state of the chain of ^{40}Ca ions and construct outer product state
- Construct operator U_A that describes a π pulse directed to ion A with energy $\hbar(\omega_o - \omega_Z)$
- The laser pulse generates Rabi oscillations
- Ion A generates phase *–i, changes ion A from |1>→|0> and phonon vibrational state |0>→|1> (ion B unaffected)*

$$|0_A> |0_B> |0> \longrightarrow |0_A> |0_B> |0>$$

$$|0_A> |1_B> |0> \longrightarrow |0_A> |1_B> |0>$$

$$|1_A> |0_B> |0> \longrightarrow -i\,|0_A> |0_B> |1>$$

$$|1_A> |1_B> |0> \longrightarrow -i\,|0_A> |1_B> |1>$$

Generate Laser Pulse Directed to Ion B

- Construct operator V_B that generates a π pulse directed to ion B and changes the phase of the wavefunction by π
- Occurs only if ion B is the ground state |0> and the phonons are in excited vibrational state |1>

$$|0_A>|0_B>|0> \longrightarrow |0_A>|0_B>|0>$$

$$|0_A>|1_B>|0> \longrightarrow |0_A>|1_B>|0>$$

$$-i\,|0_A>|0_B>|1> \longrightarrow i\,|0_A>|0_B>|1>$$

$$-i\,|0_A>|1_B>|1> \longrightarrow -i\,|0_A>|1_B>|1>$$

Apply Operator U_A a Second Time with a π Pulse Directed to Ion A

- π pulse again directed to ion A
- If ion A is in state |0> generates a phase rotation of –*i* and changes the state of ion A from |0>→|1> and the vibrational phonon state from |1>→|0>

$$|0_A>|0_B>|0> \longrightarrow |0_A>|0_B>|0>$$

$$|0_A>|1_B>|0> \longrightarrow |0_A>|1_B>|0>$$

$$i\,|0_A>|0_B>|1> \longrightarrow 1\,|1_A>|0_B>|0>$$

$$-i\,|0_A>|1_B>|1> \longrightarrow -\,|1_A>|1_B>|0>$$

Construct a 2 Qubit Truth Table for the Product Operation $W=U_A V_B U_A$

Initial State

$$|0_A> |0_B> |0>$$
$$|0_A> |1_B> |0>$$
$$|1_A> |0_B> |0>$$
$$|1_A> |1_B> |0>$$

Final State

$$|0_A> |0_B> |0>$$
$$|0_A> |1_B> |0>$$
$$|1_A> |0_B> |0>$$
$$-|1_A> |1_B> |0>$$

$$W_{CPHASE} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Recall The Property of a Control Phase Gate

- In a $\binom{1}{0}, \binom{0}{1}$ basis, the Control Phase gate changes the sign of the 2nd qubit when the 1st qubit is 1

$$W_{CPHASE} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

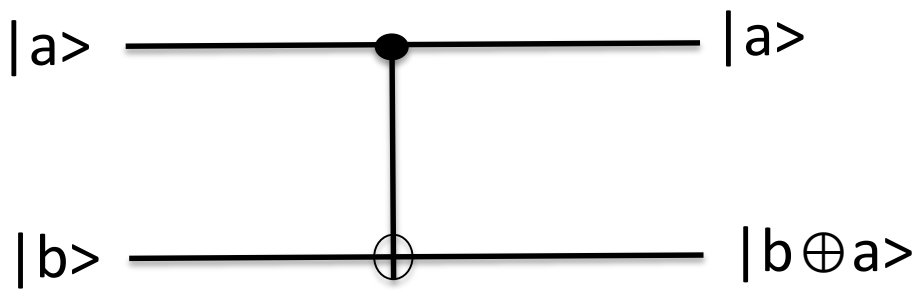
$$W_{CPHASE}^{\dagger} W_{CPHASE} = I$$

- The CPHASE gate becomes a CNOT universal quantum gate when combined with 2 Hadamard gates

Recall the Property of the CNOT Gate

Matrix representation of the CNOT gate

$$U_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad U_{CNOT}^{\dagger} U_{CNOT} = I$$

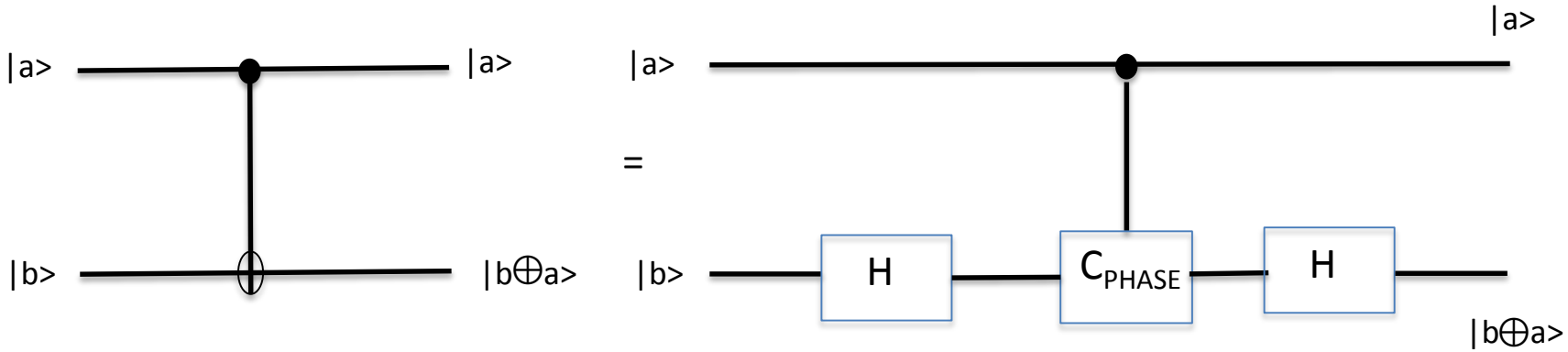


$|a\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $|b\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

|aa> → |aa>
|ab> → |ab>

|ba> → |bb>
|bb> → |ba>

Express CNOT in Terms of CPHASE



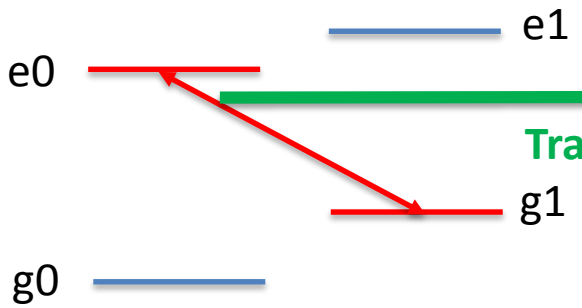
Operation of the Trapped Ion Quantum Computer

1. Select any two ions ("A" and "B") in the 1-dim chain of ^{40}Ca
2. Generate a laser pulse to force ion "B" into an up spin state
3. If the ion B is spin up use another laser pulse to induce center of mass motion of the ion chain (common dipole motion)
4. The center of mass (CoM) motion is uniformly detected everywhere along the ion chain
5. Swap the information from the up state of ion B to the center of mass motion of the ion chain (essentially communicate signal on the "data bus" of ion chain that the ion "B" is spin up)

Information SWAP Between Ion Spin State and Phonon Center of Mass Vibrational State

- g_0 and e_0 are the internal states of the ion
- Construct arbitrary qubit state $(\alpha|g> +\beta|e>)$ with the center of mass motion |0> laser cooled to ground state
- Fire another π pulse this time between states |e0> and |g1>
- Probability amplitudes α and β transferred from the internal spin state of the ion to the phonon vibrational center of mass state

$$(\alpha|g> +\beta|e>)|0>= \alpha|g0> +\beta|e0>$$



Transfer information from |e0> to g1>

$$= \alpha|g0> +\beta|g1>$$

$$\alpha|g0> +\beta|g1>= |g> (\alpha|0> +\beta|1>)$$

Information now in phonon state that is center of mass motion

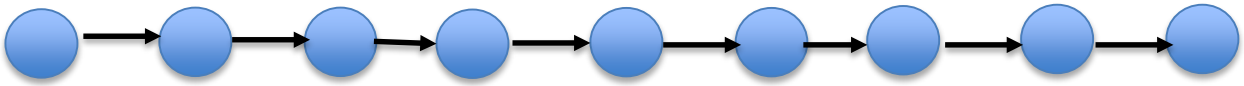
Operation of the Trapped Ion Quantum Computer

6. Communicate Ion B information to Ion A by constructing a phase gate via the data bus (CoM motion of phonons)
7. Change rotation of the wavefunction but only if both ions are spin up
8. Replace the information on the data bus back into the original Ion B (this clears the data bus)
9. Now have a measurement of Ion A's state without disturbing it in a way that collapses the entire TIQC state wavefunction
10. Quantum computation can continue to next gate operation

Step 3
Ability to Extract a Final Measurement From The State of the Qubits at the Conclusion of the Quantum Computing Program

Can Now Construct Quantum Computer

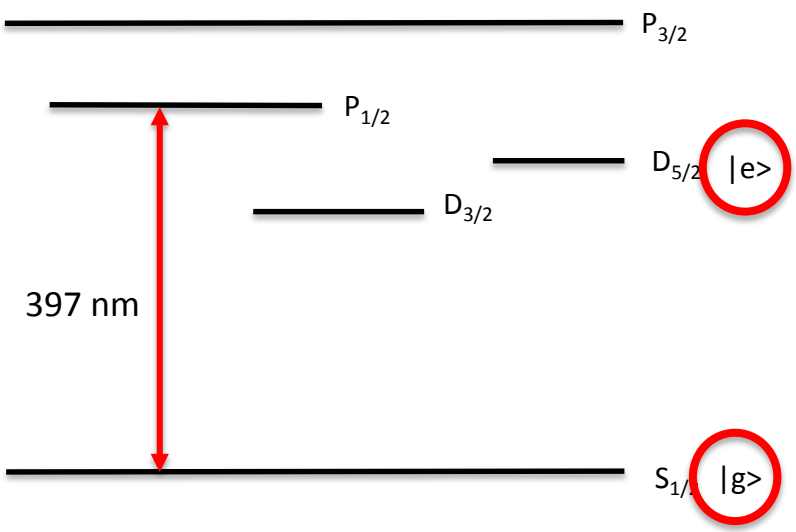
- These 2 ions form quantum computing 2 qubit operations
- Can operate a TIQC with many ions that provide 2^N states using combinations or any 2 ions remotely separated from each other



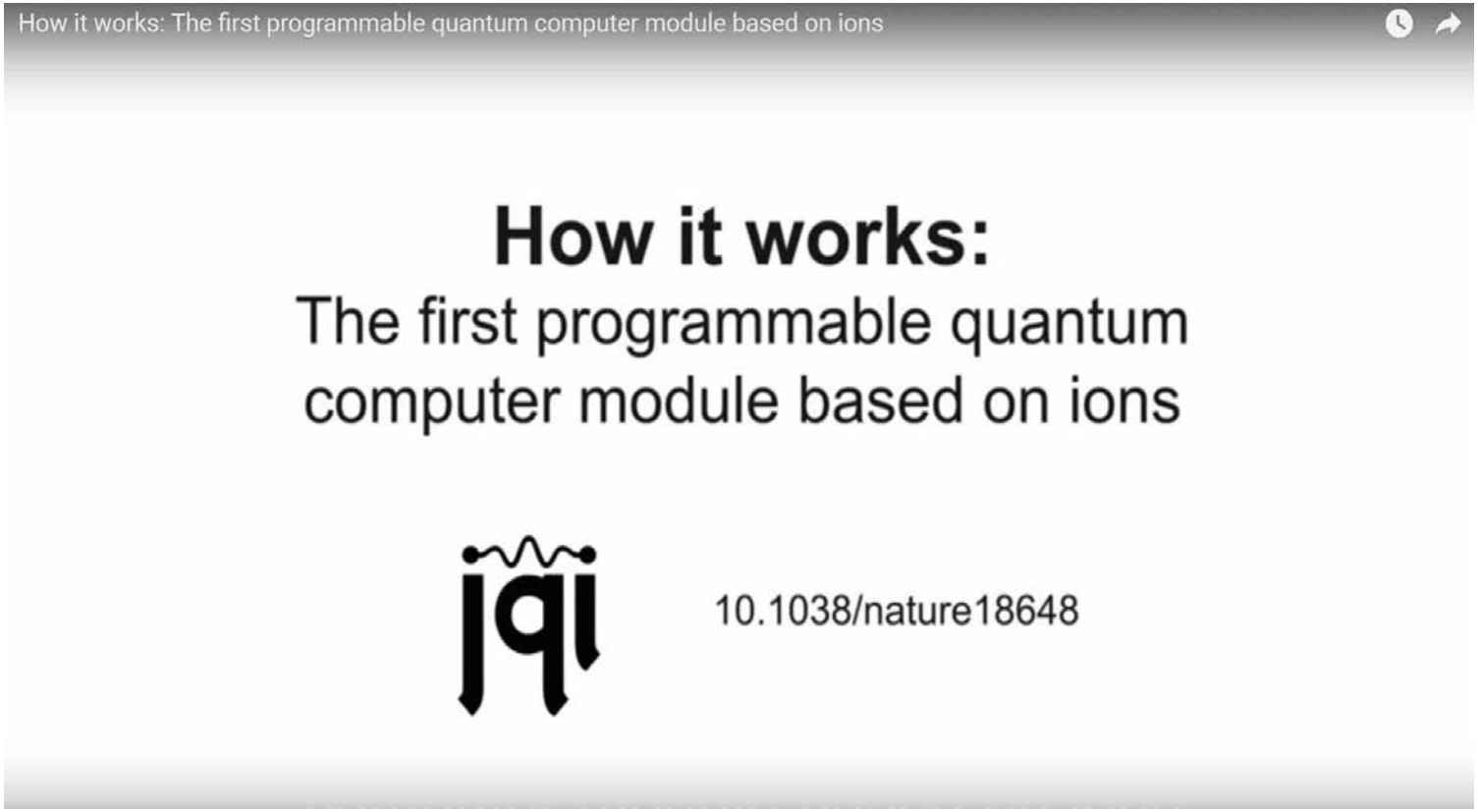
- This procedure selects only the 2 ions that participate in the interaction while all other ions in the chain are undisturbed (no measurement disturbance of the wavefunction)

Measuring the Final State of the Two Qubit System for the $^{40}Ca^{+}$ Trapped Ion Quantum Computer

- Measurement is done using the 397 nm laser to detect whether or not there is fluorescence between the $P_{1/2} \rightarrow S_{1/2}$ transition
- If the ion is in the ground state ("0" state) then the ion will fluoresce and a 397 nm light signal will be observed
- If the ion is in the $D_{5/2}$ state ("1" state) there will be no fluoresce at 397 nm and no light signal will be observed



Ion Trap Quantum Computer Simulation*



* https://www.youtube.com/watch?v=eK6g6ozLcVA

Step 4
System Must Be Scalable

Comments - Ongoing Research Criteria 5 - System Must Be Scalable

- TIQC requires very pure state initialized which implies very low (milli-Kelvin) operating temperature for the apparatus
- The frequency of the data bus must be slower than the frequency of the center of mass phonon vibrational mode
- As the number of ions increases the difficulty of maintaining a coherent state wavefunction also increases (ex. stray external EM fields) – increasing likelihood of a destroying the coherence and leaving a collapsed wavefunction before the completion of the full set of gate operations
- Ongoing work to improve the performance and operation of TIQC devices

Last Slide